ANALOG DEVICES

Universal Trigonometric Function Converter

AD639*

FEATURES
Complete, Fully-Calibrated Synthesis System
All Standard Functions: Sin, Cos, Tan, Cosec,
 Sec, Cot, Arcsin, Arccos, Arctan, etc.
Accurate Law Conformance (Sine to 0.02%)
Angular Range of ±500° (Sine Mode)
Function Programmable by Pin Strapping
1.5MHz Bandwidth (Sine Mode)
Multiplication via External Amplitude Input

APPLICATIONS
Continuous Wave Sine Generators
Synchro Sine/Cosine Multiplication
Coordinate Conversion and Vector Resolution
Imaging and Scanning Linearization Circuits
Quadrature and Variable Phase Oscillators

FUNCTIONAL BLOCK DIAGRAM

PRODUCT DESCRIPTION
The AD639 is a high accuracy monolithic function converter which provides all the standard trigonometric functions and their inverses via pin-strapping. Law conformance and total harmonic distortion surpass that previously attained using analog shaping techniques. Speed also exceeds that possible using ROM look-up tables and a DAC; in the sine mode, bandwidth is typically 1.5MHz. Unlike other function synthesis circuits, the AD639 provides a smooth and continuous sine conformance over a range of −500° to +500°. A unique sine generation technique results in 0.02% law conformance errors and distortion levels of −74dB in triwave to sinewave conversion.

The AD639 is available in three performance grades. The A and B are specified from −25°C to +85°C and the S is guaranteed over the extended temperature range of −55°C to +125°C. All versions are packaged in a hermetic TO-116, 16-pin ceramic DIP. A-grade chips are also available.
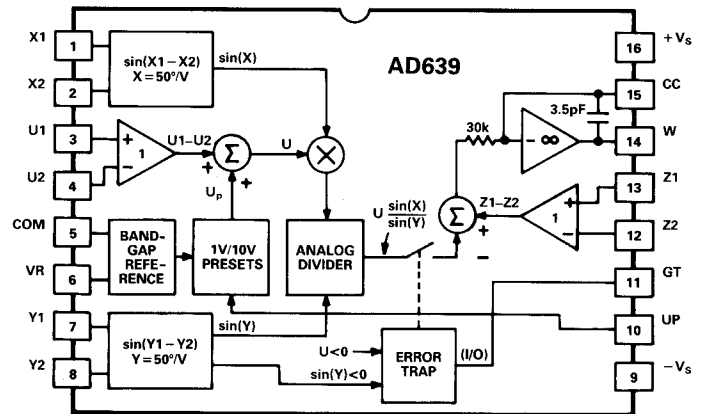
PRODUCT HIGHLIGHTS
The AD639 generates a basic function which is the ratio of a pair of independent sines:

$$W = U \frac{\sin (x_1 - x_2)}{\sin (y_1 - y_2)}$$

The differential angle arguments are proportional to the input voltages X and Y scaled by 50°/V. Using the 1.8V on-board reference any of the angular inputs can be preset to 90°. This provides the means to set up a fixed numerator or denominator (sin 90° = 1) or to convert either sine function to a cosine (cosθ = sin (90° − θ)). Using the ratio of sines, all trigonometric functions can be generated (see Table I).

The amplitude of the function is proportional to a voltage U, which is the sum of an external differential voltage ($U_1 - U_2$) and an optional internal preset voltage (U_p). The control pin UP selects a 0V, 1V or 10V laser-trimmed preset amplitude which may be used alone ($U_1 - U_2 = 0$) or internally added to the $U_1 - U_2$ analog input. At the output, a further differential voltage Z can be added to the ratio of sines to obtain the offset trigonometric functions versine (1 − cos θ), coversine (1 − sin θ) and exsecant (1 − sec θ). A gating input is available which may be used to enable or disable the analog output. This pin also acts as an error flag output in situations where a combination of inputs will cause the output to saturate or to be undefined.

In the inverse modes, the argument can be the ratio of two input signals. This allows the user to compute the phase angle between the real and imaginary components of a signal using the arctangent mode.

*Protected by U.S. Patent Numbers 3,887,863; 4,475,169; 4,476,538.

SPECIFICATIONS (typical @T$_A$=25°C, V$_S$= ± 15V, U or U$_p$= 10V unless otherwise specified)

Parameter	Conditions	AD639A Min	Typ	Max	AD639B Min	Typ	Max	AD639S Min	Typ	Max	Units
SYSTEM PERFORMANCE											
SINE AND COSINE MODE ACCURACY											
Law Conformance[1]	−90° to +90°, U = 10V		0.02			0.02			0.02		%
Total Harmonic Distortion[2]	@10kHz, U = 10V		−74			−74			−74		dB
Mismatch of Six Peaks	−540° to +540°		0.05			0.05			0.05		%
Output Noise	@10kHz, U = 10V		2.8			2.8			2.8		μV/√Hz
	@10kHz, U = 1V		0.5			0.5			0.5		μV/√Hz
PEAK ABSOLUTE ERROR											
Sine Mode	−90° to +90°, U$_p$= 10V		0.4	0.8		0.2	0.4		0.2	0.8	%FS
	T$_{min}$ to T$_{max}$		1.0			0.8	1.8		1.2	2.5	%FS
Cosine Mode	−90° to +90°, U$_p$= 10V		0.6	1.2		0.4	0.7		0.5	1.2	%FS
	T$_{min}$ to T$_{max}$		1.5			1.2	2.0		1.7	2.7	%FS
Sine or Cosine	−180° to +180°, U$_p$= 10V		0.8	1.5		0.5	0.8		0.6	1.5	%FS
	T$_{min}$ to T$_{max}$		1.7			1.3	2.5		2.1	3.0	%FS
	−360° to +360°, U$_p$= 10V		1.2			1.0			0.9		%FS
	−90° to +90°, U$_p$= 1V		1.3	2.5		1.0	1.7		0.9	2.5	%FS
	T$_{min}$ to T$_{max}$		1.5			1.0	2.3		2.0	3.5	%FS
	−180° to +180°, U$_p$= 1V		1.5	3.0		1.2	2.0		1.1	3.0	%FS
	T$_{min}$ to T$_{max}$		1.7			1.3	2.5		2.3	4.0	%FS
	−360° to +360°, U$_p$= 1V		2.0			1.8			1.5		%FS
vs Supply	−360° to +360°, U$_p$= 10V V$_s$ = ± 15V ± 1V		0.02			0.02			0.02		%FS/V
	−360° to +360°, U$_p$= 1V V$_s$ = ± 15V ± 1V		0.07			0.07			0.07		%FS/V
TANGENT MODE ACCURACY											
Peak Error[3]	−45° to +45°, U$_p$= 10V		0.5	3.5		0.5	2.0		0.5	3.5	%FS
	T$_{min}$ to T$_{max}$		2.5			1.5	2.8		3.0		%FS
	−45° to +45°, U$_p$= 1V		0.9	5.0		0.9	3.0		0.9	2.5	%FS
	T$_{min}$ to T$_{max}$		4.0			2.0	5.0		1.5	3.0	%FS
ARCTANGENT MODE ACCURACY											
Peak Angular Error											
Fixed Scale	U$_p$= 1V		0.5			0.5			0.5		Degrees
Variable Scale	U = 0.1V, −11V≤Z≤ +11V		1.5			1.5			1.5		Degrees
	U = 10V, −11V≤Z≤ +11V		0.2			0.2			0.2		Degrees
SECTIONAL SPECIFICATIONS											
ANGLE INPUTS (X1 & X2, Y1 & Y2)[4]											
Input Resistance to COM			3.6			3.6			3.6		kΩ
Nominal Scaling Factor			50			50			50		°/V
X1 & X2 Inputs											
Angular Range For Specified Error(X1 − X2)		−360		+360	−360		+360	−360		+360	Degrees
Scaling Error X1 or X2			0.2	0.65		0.2	0.65		0.2	0.65	%
Angular Offset X1 = X2 = 0			0.1	0.3		0.1	0.3		0.1	0.3	Degrees
Y1 & Y2 Inputs											
Angular Range For Specified Error (Y1 − Y2)		0		+180	0		+180	0		+180	Degrees
Scaling Error Y1 or Y2			0.2	2.0		0.2	2.0		0.2	1.0	%
Angular Offset Y1 = Y2 = 0			0.1	1.0		0.1	1.0		0.1	0.5	Degrees
AMPLITUDE INPUTS (U1 & U2)											
Input Resistance to COM			50			50			50		kΩ
Nominal Gain	X = Y = VR, W to Z1		1			1			1		V/V
Gain Error	U = 0.1 to 10V		0.01	0.5		0.01	0.5		0.01	0.5	%
	T$_{min}$ to T$_{max}$		0.08			0.08			0.25		%
Voltage Offset	U$_1$ = U$_2$ = 0V		3.0	10		3.0	10		3.0	10	mV
	T$_{min}$ to T$_{max}$		3.0			3.0			4.0		mV
Linearity Error	0≤U$_1$ − U$_2$≤10V		0.1			0.1			0.1		%
AMPLITUDE PRESET (UP)											
1V Preset Enabled	UP tied to −V$_S$										
Amplitude Accuracy			0.4	2.0		0.4	2.0		0.4	2.0	%
	T$_{min}$ to T$_{max}$		1.5			1.5			2.0		%
10V Preset Enabled	UP tied to +V$_S$										
Amplitude Accuracy			0.1	0.55		0.1	0.55		0.1	0.55	%
	T$_{min}$ to T$_{max}$		1.0			1.0			1.5		%
INVERSE INPUTS (Z1 & Z2)											
Input Resistance to COM			50			50			50		kΩ

| Parameter	Conditions	AD639A Min	AD639A Typ	AD639A Max	AD639B Min	AD639B Typ	AD639B Max	AD639S Min	AD639S Typ	AD639S Max	Units
SIGNAL OUTPUT (W)[5]	$R_L \geq 2k\Omega, C_L \leq 100pF$										
Small Signal Bandwidth W to Z1											
	$Cc = 0$		1.5			1.5			1.5		MHz
	$Cc = 200pF$		30			30			30		kHz
Slew Rate	$Cc = 0$		30			30			30		V/μs
Output Voltage Swing		± 11	± 13		± 11	± 13		± 11	± 13		V
Short Circuit Current		20	30	45	20	30	45	20	30	45	mA
Output Offset	$Z_1 = Z_2 = 0, U_p = 10$		5	30		5	30		5	30	mV
	T_{min} to T_{max}		10			10					mV
	$Z_1 = Z_2 = 0, U_p = 1V$			20			20			20	mV
	T_{min} to T_{max}		7			7					mV
VOLTAGE REFERENCE (VR)											
$R_L \geq = 1.8k\Omega$											
Nominal Output			+1.8			+1.8			+1.8		V
Output Voltage Tolerance			0.05	0.45		0.05	0.45		0.05	0.45	%
	T_{min} to T_{max}		0.08			0.08	0.5		0.2	0.6	%
Supply Regulation	$+V_S = 5V$ to $18V$		150			150			150		μV/V
Maximum Output Current			4			4			4		mA
GATE I/O (GT)											
Switching Threshold as an Input	Output Valid		+1.5			+1.5			+1.5		V
	Output Invalid		0.1			0.1			0.1		V
Voltage Output	Error, $R_L = 5k\Omega$		+2.25			+2.25			+2.25		V
	No Error, $R_L = 5k\Omega$		−0.25			−0.25			−0.25		V
POWER SUPPLIES											
Operating Range		± 5.5		± 18	± 5.5		± 18	± 5.5		± 18	V
$+V_S$ Quiescent Current	$U = X = 0V, Y = Vr$		8.0	11		8.0	11		8.0	11	mA
$-V_S$, Quiescent Current	$U = X = 0V, Y = Vr$		5.5	7.5		5.5	7.5		5.5	7.5	mA
TEMPERATURE RANGE											
Operating, Rated Performance		−25		+85	−25		+85	−55		+125	°C
Storage		−65		+150	−65		+150	−65		+150	°C
PACKAGE OPTION											
16-Pin Ceramic Side Braze DIP (D)			AD639AD			AD639BD			AD639SD		
									AD639SD/883B		
Chips			AD639 A-Chips								

NOTES

[1]Intrinsic accuracy measured at an amplitude of 10V using external adjustments to absorb residual errors in angular scaling, angular offset, amplitude scaling and output offset.

[2]Using a time and amplitude symmetric triangular wave of + 3.6V peak-to-peak and external adjustments to absorb residual errors in angular scaling and offset.

[3]Full scale is defined as the ideal output when the angle input is at either end of the limit specified.

[4]Specifications for the X inputs apply for range U = 1V to 10V, while the Y input errors are specifically given for U = 1V.

[5]When driving loads of less than 4kΩ, a 25pF capacitor from pin 15 to pin 9 avoids possible instability, although this is unnecessary when C_L is greater than 150pF.

Specifications subject to change without notice.

Specifications shown in **boldface** are tested on all production units at final electrical test. Results from those tests are used to calculate outgoing quality levels.

All min and max specifications are guaranteed, although only those shown in boldface are tested on all production units. Contact the factory for details.

PIN CONFIGURATION
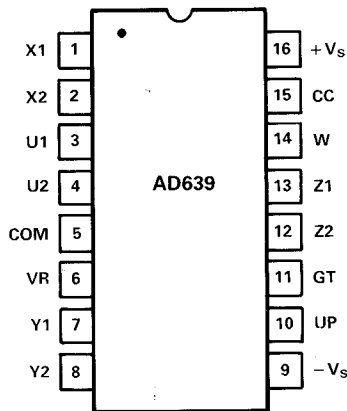
METALIZATION PHOTO
CHIP DIMENSIONS AND BONDING DIAGRAM

Dimensions shown in inches and (mm).
Consult factory for latest dimensions.
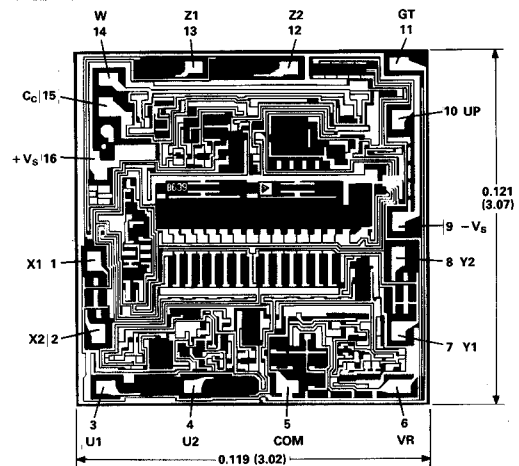
ABSOLUTE MAXIMUM RATINGS

	AD639A,B	AD639S
Supply Voltage	±18V	★
Internal Power Dissipation	300mW	★
Output Short-Circuit to Ground	Indefinite	★
Input Voltages X_1, X_2, Y_1, Y_2[1]	±12V	★
Input Voltages U_p, U_1, U_2, Z_1, Z_2[1]	±25V	★
Operating Temperature Range	−25°C to +85°C	−55°C to +125°C
Storage Temperature Range	−65°C to +150°C	★
Lead Temperature, Soldering	60sec, +300°C	★
Thermal Resistance, θ_{JC}	22°C/W	★
Thermal Resistance, θ_{JA}	90°C/W	★

NOTES
★Same as AD639A,B Specifications
[1]These inputs are purely resistive and the maximum inputs are determined by resistor dissipation limits, not the supply voltages.

$\sin(\theta) = \dfrac{\sin(\theta)}{1} = \dfrac{\sin(\theta-0)}{\sin(90°-0)}$	$\operatorname{cosec}(\theta) = \dfrac{1}{\sin(\theta)} = \dfrac{\sin(90°-0)}{\sin(\theta-0)}$	
$\cos(\theta) = \dfrac{\cos(\theta)}{1} = \dfrac{\sin(90°-\theta)}{\sin(90°-0)}$	$\sec(\theta) = \dfrac{1}{\cos(\theta)} = \dfrac{\sin(90°-0)}{\sin(90°-\theta)}$	
$\tan(\theta) = \dfrac{\sin(\theta)}{\cos(\theta)} = \dfrac{\sin(\theta-0)}{\sin(90°-\theta)}$	$\operatorname{cotan}(\theta) = \dfrac{\cos(\theta)}{\sin(\theta)} = \dfrac{\sin(90°-\theta)}{\sin(\theta-0)}$	

Table I.

Principles Of Operation

Figure 1 is a functional equivalent of the AD639, intended to assist in understanding and utilizing the device: it is not a literal representation of the internal circuitry[1]. Two similar sine-shaping networks accept input voltages X_1, X_2, Y_1 and Y_2, proportional to the corresponding angles x_1, x_2, y_1 and y_2, with a scaling factor of 50°/V (20mV/°).
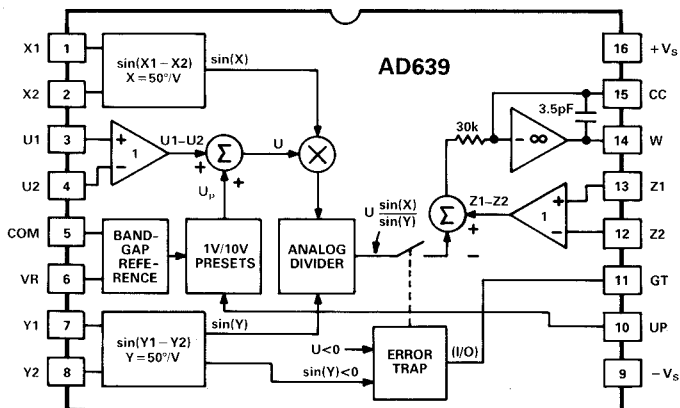

Figure 1. Equivalent Block Schematic of the AD639

The first of these networks generates an output proportional to the sine of $x = (x_1 - x_2)$ over a useful operating range in excess of −500° to +500° (see Figure 3). The accuracy of the function over the central ±180° is excellent, a consequence of the optimized network design, further enhanced by precision laser wafer trimming during manufacture. The output of the X-network is multiplied by the amplitude-control voltage, U. This may be

provided by applying inputs to U1 or U2, or pre-selected to be 1V or 10V by a control input to UP, or in combination; that is, the function amplitude is $U = (U_1 - U_2) + U_p$.

The second network generates an output proportional to the sine of $y = (y_1 - y_2)$. Although the X and Y networks are similar, other design considerations result in a smaller angular range for the Y-input. The principal range is from 0° to +180°; in the adjacent ranges (+180° to +360° and 0° to −180°) the error trap is activated.

The ratio of the two sines is generated by *implicit* division, rather than by use of a separate analog divider as indicated in Figure 1, and is summed with the voltage $Z = (Z_1 - Z_2)$. The difference is applied to the high-gain output op-amp. In the *normal* modes (see below) Z1 is connected to the output W, and Z2 is grounded. Under these conditions, the function is

$$W = U\,\frac{\sin(x_1 - x_2)}{\sin(y_1 - y_2)}$$

Either sine function can be converted to the cosine by applying the input to X2 or Y2 and introducing a +90° offset, since

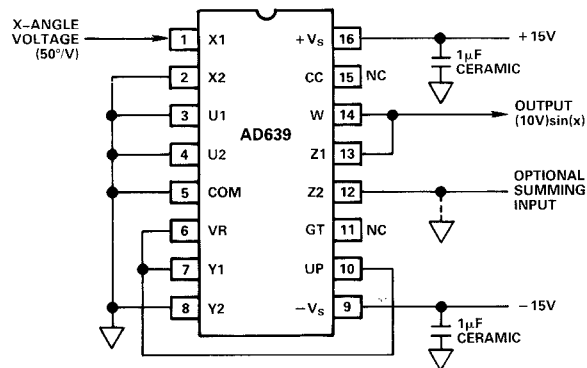*Figure 2. Connections for the Sine Mode with Amplitude Preset to 10V*

[1]For details of the sine-network theory and design, see "A Monolithic Microsystem for Analog Synthesis of Trigonometric Functions and their Inverses," Barrie Gilbert, *IEEE Journal of Solid-State Circuits*, Vol. SC-17, No. 6, Dec. 1982, pp 1179-1191. Reprints available.

$\cos(\theta) = \sin(90° - \theta)$. For example, by connecting the $+1.8V$ reference output at pin 6 (VR) to X1 and the angle voltage, V_θ, to X2 the numerator becomes the cosine of angle θ. Alternatively, by connecting VR to either X1 or Y1 and grounding X2 or Y2, the numerator or denominator, respectively, becomes unity, since $\sin(90° - 0) = 1$. By these means, the full set of *normal* functions shown in Table I can be generated. All functions can be sign-inverted by interchanging the X-inputs. The Z2 input can be used to sum another function to the output, W, with unity gain.

In addition to the *normal* modes providing sine, cosine, tangent, cosecant, secant and cotangent functions, the AD639 can generate the *offset* functions such as the versine, $1 - \cos(\theta)$, discussed below. The *inverse* functions such as arc-sine, arc-cosine and arc-tangent, are also supported by the AD639, by closing the feedback loop through the corresponding *normal* function. The output angle is limited to the principal range (for example, $-90°$ to $+90°$ for the arc-sine and arc-tangent, $0°$ to $+180°$ or $-180°$ to $0°$ for the arc-cosine).

TERMINOLOGY

When discussing a device having as many inputs and operating modes as the AD639, it is important to clarify the nomenclature and scaling conventions. In all cases *angles* are denoted by lower-case letters (x, y, θ) and have the dimension of angular degrees. Upper-case letters (A, V, U, W, X, Y and Z) refer to *voltages*; subscripts are used to refer to one or the other of a differential pair such as $X_1 - X_2$, or the preset value U_p. Numbered upper-case letters refer to the variable name or the package pin.

THE ANGLE INPUTS: X1, X2, Y1, Y2

The angles $x = (x_1 - x_2)$ and $y = (y_1 - y_2)$ are directly proportional to the differential voltages $X = (X_1 - X_2)$ and $Y = (Y_1 - Y_2)$ respectively, with a scaling factor of 50°/V. The X-inputs can be driven to $\pm12V$ pk, that is $\pm600°$. The Y-input should be limited to 0 to $+3.6V$ (0° to $+180°$) to satisfy certain internal requirements. The resistance at these inputs is nominally 3.6kΩ to COM.

The sine function exhibits odd-order symmetry: $\sin(-\theta) = -\sin(\theta)$. By simply interchanging the X-inputs, the overall sign of any function can be inverted. The Y-inputs can also be interchanged to allow operation with a negative input voltage (0 to $-3.6V$) while maintaining the correct angular range.

It may occasionally be desirable to reduce the angular scaling factor. For example, to convert a triwave of $\pm10V$ amplitude into a continuous sinewave requires a scaling factor of 9°/V (since $\pm10V$ corresponds to $\pm90°$). This can be achieved by using a resistor (in this case, about 16.4kΩ) in series with the X1 input; a resistor of equal value must be inserted in series with the X2 input to minimize angular offset error. Note that the on-chip thin-film resistors are not trimmed to absolute value, so a scaling adjustment is needed; however, once set, scaling will be stable.

THE AMPLITUDE-CONTROL INPUTS: U1, U2, UP

The amplitude of the function can be determined either by the application of an external voltage to the U1 and U2 inputs, or by enabling the internal preset voltage U_p by taking the control pin UP low or high, or via a combination of these modes. The net amplitude is $U = (U_1 - U_2) + U_p$. This sum must be greater than zero and less than $|-V_s|$; voltages beyond these limits activate the error trap.

In the external mode, the differential voltage $(U_1 - U_2)$ will generally be in the range 10mV to 10V. Positive inputs are applied to U1 while U2 is grounded; for negative inputs, interchange U1 and U2. The input resistance at U1 and U2 is nominally 50kΩ to analog common. A nominal bias current of $-50\mu A$ is needed at the U-inputs; zero-valued inputs must therefore be connected to common to prevent offset error. The gain from the U-interface to the output is trimmed to be unity for $\sin(x)/\sin(y) = 1$. The effective gain can be lowered using a series resistor; to avoid offset an equal resistor must be used in the zero-valued input.

The UP control pin may be left unconnected (or grounded) to disable the internal amplitude preset, connected to $+V_s$ to set $U_p = 10V$, or to $-V_s$ to set $U_p = 1V$. An external resistor of 75kΩ ($\pm V_s = 5V$) to 360kΩ ($\pm V_s = 15V$) can be inserted in series with UP (which also has an input resistance of typically 50kΩ) to minimize power dissipation. Alternatively, V_r can be used to enable $U_p = 10V$ for ambient temperatures below $+60°C$. The UP input can be used to switch the output on or off under logic control, but requires a relatively long response time. The GT interface is more suitable for this purpose and it allows gating to any amplitude, U not just to the preset values of 1V or 10V.

THE REFERENCE OUTPUT: VR

The voltage V_r is laser-trimmed to $+1.8V$ with respect to analog common. It can be used to fix the angle x or y to 90° and thus set $\sin(x)$ or $\sin(y)$ to unity. It can also provide a 90° offset to convert the numerator or denominator to a cosine function. Stable offsets less than 90° may be introduced using a voltage-dividing series-resistor (nominally 3.6kΩ for 45°). V_r can also be used as the amplitude input voltage $U_1 - U_2$, or as a convenient control input to set $U_p = 10V$ for ambient temperatures below $+60°C$. This output is short-circuit protected and can provide up to 4mA total load current.

THE ERROR-TRAP AND GATE: GT

In some applications it may be useful to know that the output is severely in error due to a dynamic combination of inputs. For example, the tangent, cotangent, secant and cosecant all exhibit regions where the function increases sharply for small angular changes, and the output may easily saturate. Consider the case where (10V) $\tan(\theta)$ is being generated. W is 10V for $\theta = 45°$, and the theoretical output of 17.3V at $\theta = 60°$ cannot be achieved using $\pm15V$ supplies. Likewise, the output is invalid whenever the angle y is outside of a valid range (principally 0 to $+180°$), or when $U<0$ or $U>|-V_s|$. Under such conditions the AD639 generates a HIGH output at pin 11 and simultaneously clamps the analog output to zero (in fact, to the voltage Z_2). Grounding GT disables the error trap.

The GT pin may also be used as an *input* to gate the function output. This is achieved by raising pin 11 to a voltage above $+1.5V$. Response time is typically 500ns for a logic drive of 0 to $+2V$, and the ON/OFF ratio is greater than 83dB when used as a continuous-wave sine converter with a single-sided $\pm1.8V$ triwave drive at frequencies up to 10kHz, or 63dB at 100kHz; the feedthrough is entirely capacitive, and is equivalent to 5pF between X1 or X2 and the op-amp summing node. Feedthrough can be minimized by using a balanced drive to X1 and X2.

Operation in Normal Modes

In *normal* modes, the Z-input establishes a feedback path around the output op-amp, by connecting Z1 to the output, W, and Z2 to the ground associated with the load circuit. For the highest accuracy Z1 can be used to sense the output at the load terminals. Similarly, zero-valued angle inputs and the angle common (pin 5) should be connected to the ground associated with the source circuitry.

SINE MODE

The AD639 can generate either (1) a low-distortion continuous sinewave from a repetitive triwave input or (2) a high-accuracy sine function for use in computational applications. In most cases, the choice of preset or externally-controlled amplitude will make little difference to distortion or accuracy, and both methods are used in this section. In all of the *normal* modes, the Z2 input can be used either to sum a further signal to the output (or introduce an optional output offset trim). The denominator is set to unity by making $y = 90°$, using the $+1.8V$ output. Figure 2 shows typical connections. The 10V preset is selected, using V_r as a control input to UP, and the ideal output is $(10V)\sin(x)$. In practice, five basic types of error arise:

1. *X-angle scaling error*: The amount by which the angle generated for each volt of X-input differs from 50°. In triwave-to-sinewave (CW) applications this introduces odd-order harmonic distortion, and is indistinguishable from an incorrect triwave amplitude.

2. *X-angle offset error*: The actual angle generated when $X = (X_1 - X_2) = 0$. In CW applications this introduces even-order harmonic distortion, as a non-zero mean in the triwave would.

3. *Amplitude scaling error*: The amount by which the peak-to-peak amplitude of the sinewave differs from the ideal value, $U/\sin(y)$. This error is usually critical only in computational applications. Errors associated with the Y-network also affect the amplitude in the sine mode.

4. *Output offset error*: The amount by which the *mean value* of the sinewave differs from zero (strictly, the voltage on Z2). This error is only important in computational applications. Note that the output may also be non-zero due to angular offset on the X-input. For example, the typical specified X-angle offset of 0.1° introduces an output error of 17.45mV when $U/\sin(y) = 10V$, more than three times the specified mean offset component of 5mV.

5. *Law-conformance error*: The residual deviation between the output function and the ideal function when all of the above errors have been removed by trimming during manufacture or further external trimming, limiting the ultimate accuracy of the function.

Figure 3 shows the function when driven well beyond the specified angular range, using a differential X-input of $\pm 18V$ peak. This also shows the AD639's ability to drive $\pm 15V$ into a 600Ω load, with supplies of $\pm 18V$. Using an accurate data-acquisition system the output can be compared to a computer-generated sine function. When the first four types of errors are trimmed out, the peak error over the full input range is typically less than 0.5%. Over the central $-90°$ to $+90°$, the peak law-conformance error is typically only 0.02%. Figure 4 shows the law conformance for four typical samples of AD639. The differential signal interfaces simplify the inclusion of optional offset correction to any of the variables.
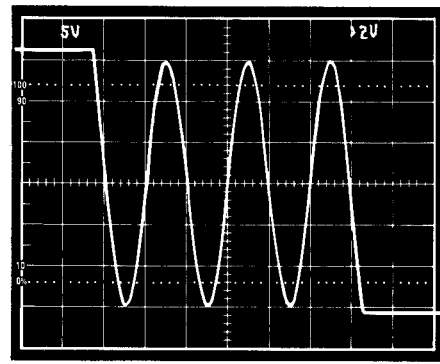

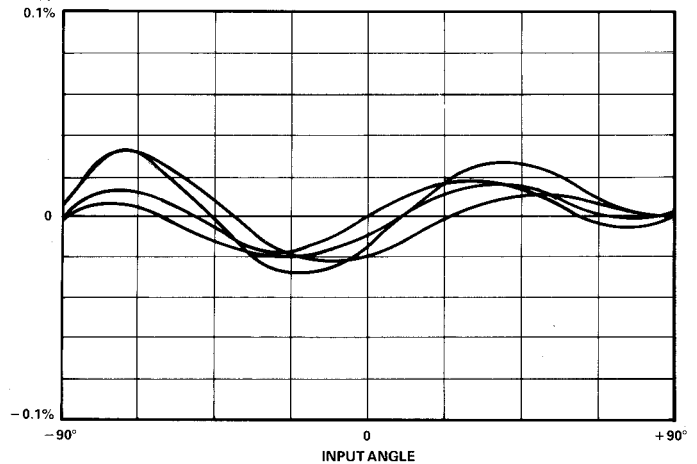Figure 3. Output Function for Peak X-Input of $\pm 18V$, with $U = 15V$, $R_L = 600\Omega$ $(\pm V_S = 18V)$

Figure 4. Residual Error Over Central 180° Using External Trimming

HARMONIC DISTORTION

The AD639 can generate continuous sinewaves of very low distortion using a linear, highly-symmetric triangle-wave of $\pm 1.8V$ amplitude. Imperfections in the triwave will cause the following errors:

1. *Incorrect amplitude*: This causes odd-order distortion. Each 1% error (either too large or too small) generates 0.25% of HD3, 0.0833% of HD5 and a total harmonic distortion (THD) of 0.27% $(-51.42dBs)$.

2. *Baseline offset*: This causes even-order distortion. Each millivolt of offset in a 1.8V triwave generates 0.037% of HD2, 0.0074% of HD4 and a THD of 0.038%, as well as a DC offset of 0.055% of the output amplitude.

3. *Time-asymmetry*: The run-up time, t_1, and run-down time, t_2, of the triwave may be unequal. This causes both odd- and even-order harmonics. Let the asymmetry in percent be $p = 100(t_1 - t_2)/(t_1 + t_2)$. The even-order terms are proportional to p; the odd-order terms increase as p^2. A 1% time-asymmetry generates 0.57% of HD2, 0.00625% of HD3, 0.043% of HD4 and 0.00167% of HD5, and a THD of $-44dBs$. There is no DC term.

4. *Amplitude-nonlinearity*: This can take on many forms, such as an exponential nonlinearity in the triwave, amplitude compression, and so on. Distortion can be calculated for various special cases. Fortunately, it is fairly easy to avoid these types of imperfections in the triwave generator using appropriate design methods.

When triwave errors are minimized, harmonic distortion can be as low as 0.01%. Figure 5 shows the output spectrum at 10kHz, with an output amplitude of 20V pk-pk and a load resistance of 10kΩ. An HP3325A synthesizer/function generator was used to produce the triwave. Distortion rises only slightly when using the minimum specified load of 2kΩ; in fact, the AD639 can drive loads down to 600Ω. At ±V$_s$= ±18V, sine amplitudes of ±15V (10.6V rms, or 225mW of load power) can be generated, with typically 0.03% HD2 and HD3.
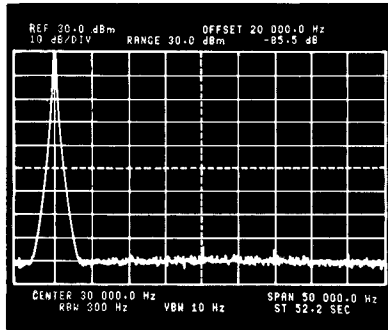
Figure 5. Spectrum of 10V Sine Output at 10kHz; HD2 = −88dBs, HD3 = −85.5dBs

COSINE MODE

The cosine function is generated by offsetting the sine by 90° using V$_r$. The X-input is connected to X2 and VR to X1; then

$$W = U\,\frac{\sin\,(90° - x)}{\sin\,(90° - 0)} = U\cos\,(x)$$

Connections for the cosine are shown in Figure 6; the amplitude in this case is determined externally, by way of illustration. The angular range now extends from −400° to +600°, with highest accuracy between 0° and +180°.

TANGENT AND COTANGENT MODES

The tangent function is provided by the connections shown in Figure 7. The angle voltage, corresponding to θ, is applied both to the numerator, set to the sine mode, and the denominator, set to the cosine of the same angle:

$$W = U\,\frac{\sin\,(θ - 0)}{\sin\,(90° - θ)} = U\,\frac{\sin\,(θ)}{\cos\,(θ)} = U\tan\,(θ)$$

Most applications require accurate operation for angles up to nearly ±90° and accordingly U is preset to 1V (rather than 10V). Under these conditions, W = 1V when θ = 45° and 11.43V when θ = 85°. Using 15V supplies, the output op-amp will be unable to generate the tangent much beyond this point: at only 86° it would theoretically need to reach 14.3V. For an input exceeding 90° in either direction the denominator becomes negative, and the error trap is enabled. Figure 8 shows the function for inputs up to ±2.5V (±125°).

The errors associated with the sine mode, (see above) apply to the tangent mode also, but the total error in the tangent, cosecant, secant and cotangent modes (when the Y-input is also varied) are higher, since the Y network is not trimmed and the angular
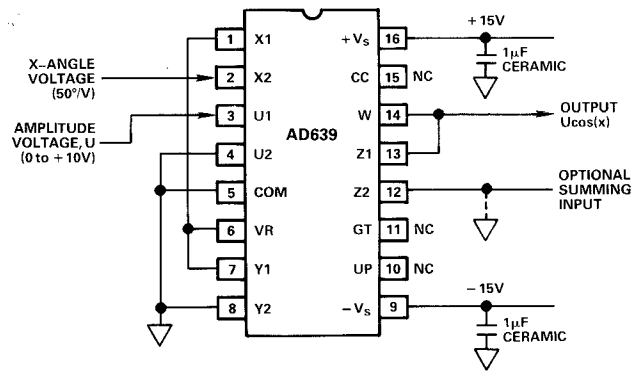

Figure 6. Connections for the Cosine Mode with External Amplitude Control
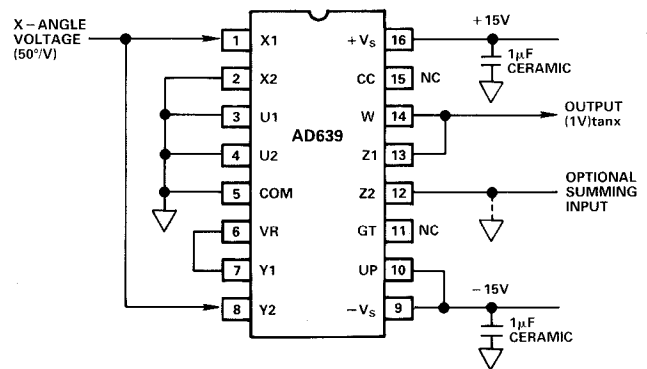

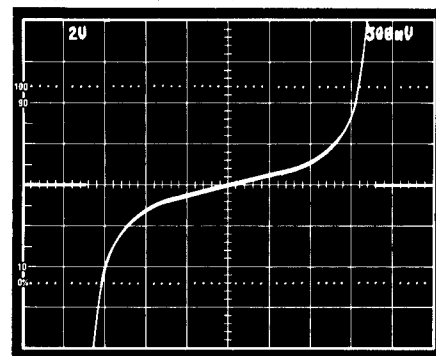Figure 7. Connections for Tangent Mode with Amplitude Preset to 1V

Figure 8. The Tangent Output for Angle Inputs Up to ±125° (Error Trap Activated Above 85°)

scaling and offset errors of this network are absorbed during trimming of the output in the sine mode.

The cotangent is generated by interchanging numerator and denominator. The principal range is now from 0 to +180°, and the output (1V)cot (θ) ranges from +11.43V at θ = 5°, through zero at θ = 90°, to −11.43V at θ = 175°.

SECANT AND COSECANT MODES

In secant and cosecant modes, the numerator is fixed to unity by connecting X1 to VR and X2 to analog common. For the secant, angle voltage A is connected to Y2 and Y1 is tied to VR; then

$$W = U \frac{\sin(90° - 0)}{\sin(90° - \theta)} = U \frac{1}{\cos(\theta)} = U \sec(\theta)$$

The principal range is $-90°$ to $+90°$. The most practical amplitude scaling is provided using the U-preset of 1V, when the output ranges from $+11.47V$ at $\theta = -85°$ and $+85°$, to $+1V$ at $\theta = 0$. The cosecant differs only slightly: the angle input is connected to Y1 and Y2 is connected to analog common, making the denominator $\sin(\theta)$. The principal range is now 0 to $+180°$. When $U = 1V$ the output is $+11.47V$ at $\theta = +5°$ and $+175°$, and $+1V$ at $\theta = 90°$.

OFFSET MODES

The versine, vers $(\theta) = 1 - \cos(\theta)$, coversine, covers $(\theta) = 1 - \sin(\theta)$, and exsecant, exsec $(\theta) = 1 - \sec(\theta)$ involve the addition of a constant term to one of the normal trigonometric functions. These can be generated with the AD639 using the Z2 input to add a voltage to the output proportional to the amplitude of the basic function. In the versine and coversine modes this is simply the same voltage as applied to U1 (U2 grounded) to set up the amplitude of the sign-inverted cosine or sine function, respectively:

$$W = U - Uf(\theta) = U(1 - f(\theta))$$

In these two modes the output starts at zero and has a peak value of twice the amplitude voltage, U.

For the exsecant a *negative* voltage is added at Z2 and this same voltage is applied to U2 with U1 grounded; this satisfies the requirement that the sign of $U_1 - U_2$ be positive. (See comments on the Amplitude Control Inputs). The angle inputs are set up for the secant; the principal range is still $-90°$ to $+90°$, but the output is now zero when the input angle is zero.

OPERATION AT LOW SUPPLY VOLTAGES

The signal ranges at the angle interfaces are essentially independent of the supply voltages. In almost all cases, the primary limitation to the function's range will arise at the output, W, which can swing to within approximately 2V of either supply. For example, the X-input may have a peak value of $\pm 12V$ ($\pm 600°$) even when using $\pm 5V$ supplies.

Inverse Function Modes

The AD639 generates the inverse trigonometric functions by closing the feedback loop around the output op-amp through the *angle* inputs, rather than through the Z-interface, resulting in a nonlinear feedback system. To understand this, note first that the *general* transfer function (with UP disabled) is

$$W = A_{OL} \left[(U_1 - U_2) \frac{\sin(x_1 - x_2)}{\sin(y_1 - y_2)} - (Z_1 - Z_2) \right]$$

where A_{OL} is the open-loop gain of the output op-amp (typically 85dB). Provided that the overall feedback remains negative, the loop can be closed in many ways, so as to force the quantity inside the brackets to a null, when

$$\frac{\sin(x_1 - x_2)}{\sin(y_1 - y_2)} = \frac{(Z_1 - Z_2)}{(U_1 - U_2)}$$

whatever combinations of variables are used to set up the feedback path. In particular, when the *angle* inputs are used the system will have one of the *normal* functions in the feedback path. The input to this system is now the *ratio* (Z/U), and the output is a voltage corresponding to the angle generated by the *inverse* of the function in the feedback path.

Since all of the normal functions are periodic, and the maximum value of the op-amp output can be equivalent to angles as large as $\pm 650°$, the closed-loop system could arrive at false "solutions" to the above equation, that is, at angles outside of the principal range. Also, the feedback can become positive in the wrong angular range, causing latch-up. Hence, it is essential to limit the magnitude of the feedback voltage. Ideally, this is done using precise active clamps, but the saturated value of the output at given supply voltages, in combination with a simple resistive divider to the angle inputs, is usually sufficient to limit operation to the principal range. The voltage at the angle inputs will be accurate, but the op-amp output will in general have inaccurate scaling and may show large offsets, due to the bias currents at the angle inputs. The error-trap should be disabled in the inverse modes by grounding GT.

ARCTANGENT MODE

The arctangent is the most useful of the inverse modes. With the connections shown in Figure 9 the loop solves the equation

$$\frac{\sin(0 - \theta)}{\sin(90° - \theta)} = -\tan(\theta) = \frac{(Z_1 - Z_2)}{(U_1 - U_2)}$$

where θ is the angle corresponding to voltage A, scaled by 50°/V. It follows that

$$\theta = \tan^{-1} \frac{(Z_2 - Z_1)}{(U_1 - U_2)}$$

The reversal of Z_1 and Z_2 in the numerator is due to the negative sign in the tangent function. The numerator may be either positive or negative, and the connections can be interchanged to alter the overall sign of the function. The denominator must be positive, but U1 and U2 may be interchanged to accept a negative input voltage. The ability of the AD639 to form the ratio of two
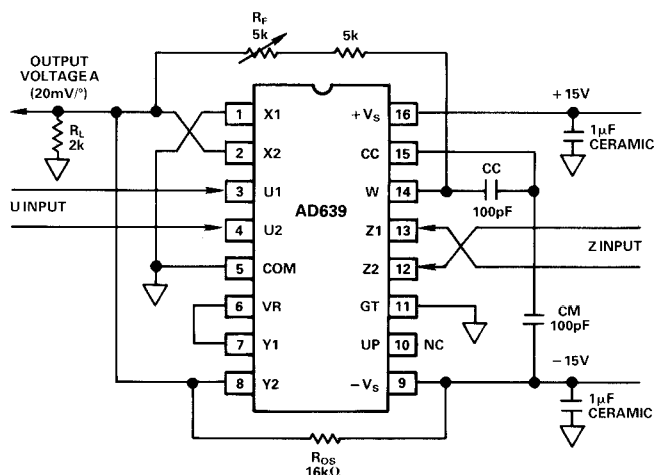

Figure 9. Connections for the General Arctangent Mode

variables prior to the arctangent operation is very useful in many applications, for example, in real-time Cartesian-to-polar conversion (see Applications section). The denominator can also be preset to 1V or 10V using the UP input; when U = 1V, the angle a is simply the arctangent of the voltage value of $Z_2 - Z_1$. Figure 10 shows an X-Y plot of the output for Z = -10V to +10V (horizontal axis of photograph) with four values of U (0.3V, 1V, 3V, 10V).
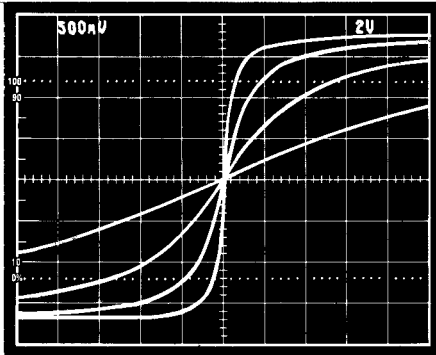

Figure 10. The Arctangent Output for Z = ±10V and U = 0.3V, 1V, 3V and 10V

Range-Limiting and Loading

Resistor R_f in Figure 9 forms a divider with the parallel sum of the input resistance at X2 and Y2 and the load resistance, shown here as 2kΩ, which prevents the output angle voltage A from exceeding ±1.7V (θ = ±85°), using ±15V supplies. This voltage is not *directly* affected by the load resistance (that is, the output behaves as a low-impedance node) but the angular range limits are. Consequently, the nominal value of R_f should be calculated for specific values of load resistance, angular range and supply voltages, and a trim range of about ±10% included to set up the angle limits correctly. R_{os} is needed to compensate the input bias currents and thus equalize the clipping limits; it does not cause an offset in θ. The direct output at pin 14 is also the arctangent but with imprecise scaling. Although this can be trimmed by R_f there will also be a supply-dependent offset due to R_{os}. For these reasons, the direct output should not be used in this mode.

HF Compensation

The output op-amp is internally compensated to be stable in all the normal modes when feedback is via the unity-gain difference amplifier associated with the Z-interface. The dominant pole is determined by the 30kΩ resistor and on-chip 3.5pF capacitor (see Figure 1) for a closed-loop bandwidth of 1.5MHz. In the arc-tangent mode, however, the gain of the feedback path is much greater than unity for practical angle values and is theoretically unbounded. For example, if the forward path is set up to generate (1V)tan(θ), the incremental gain near θ = 0 is slightly less than unity (since a 20mV change in voltage A causes a change of (1V)tan(1°) or 17.5mV in output W) but at θ = 85° the gain is 115. While the resistive divider used to limit the angle voltage A will lower the loop gain, it can still exceed unity. The capacitors C_c and C_m in Figure 9 provide the HF compensation required for operation up to ±85°, with all values of U.

ARCSINE AND ARCCOSINE MODES

The basic principles for the arcsine and arccosine are similar to those described for the arctangent. As before, the argument of the function is the ratio (-Z/U), where U may be preset to 1V or 10V, the loop gain must be negative over the principal angular range of the output, and the feedback voltage must be limited to ensure that this range is not exceeded. The loop stability is easier to ensure, since the peak gain is bounded. With U = 10V the maximum incremental gain of the forward path (at 0° for the sine and +90° for the cosine) is 8.75 and the peak loop gain is much less than this because of the attenuation used to limit the angular range. Thus relatively little additional HF compensation is required.

Connections for the arcsine are similar to the arctangent (Figure 9) except that Y2 is grounded, and C_c and C_m can be reduced or even omitted. R_f is adjusted for a peak angular range of ±90° at the (attenuated) output; if too high, the function will still be correct, but the maximum angle will be less than 90°; if too low, the function will exhibit hysteresis near the peak output. Adjustments will be needed for other values of load resistance and supply voltages. Note that the general limitation on the amplitude input (U < = |-V_s|) must be observed. Figure 11 shows an X-Y plot of the arcsine output for Z = -10V to +10V (horizontal axis of photograph) with three values of U (2V, 5V and 10V). The arcsine can be inverted by reversing the Z-interface.
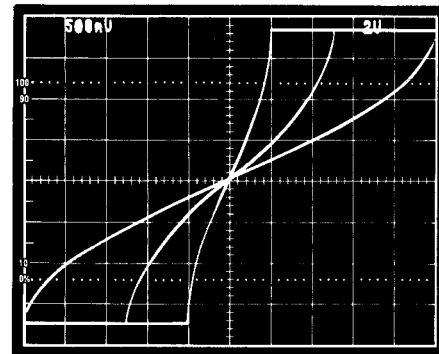

Figure 11. The Arcsine Output for Z = ±10V and U = 2V, 5V and 10V

For the arccosine, use the arcsine connections with X1 tied to V_r and insert a small-signal diode in series with R_f, having its cathode on the angle-interface side. This allows the output to move only in a positive direction. Z1 now becomes the positive numerator input, and the principal range is from 0° (when Z/U = -1) to +180°. The function is similar in appearance to the arcsine, except for the +90° output pedestal and the reversal of phase along the horizontal axis. Note that

$$\cos^{-1}(Z/U) = 90° - \sin^{-1}(Z/U) = 90° + \sin^{-1}(-Z/U).$$

To generate the negative arccosine, reverse the X- and Z-interfaces and the polarity of the diode. The output now runs from -180° for an input of Z = (Z_1 - Z_2) = -10V (with U = 10V) to 0° at Z = +10V.

It is strongly recommended that X-Y oscilloscope methods are used to investigate functional behavior during the development of any of these modes of operation: time-domain displays can easily become confusing.
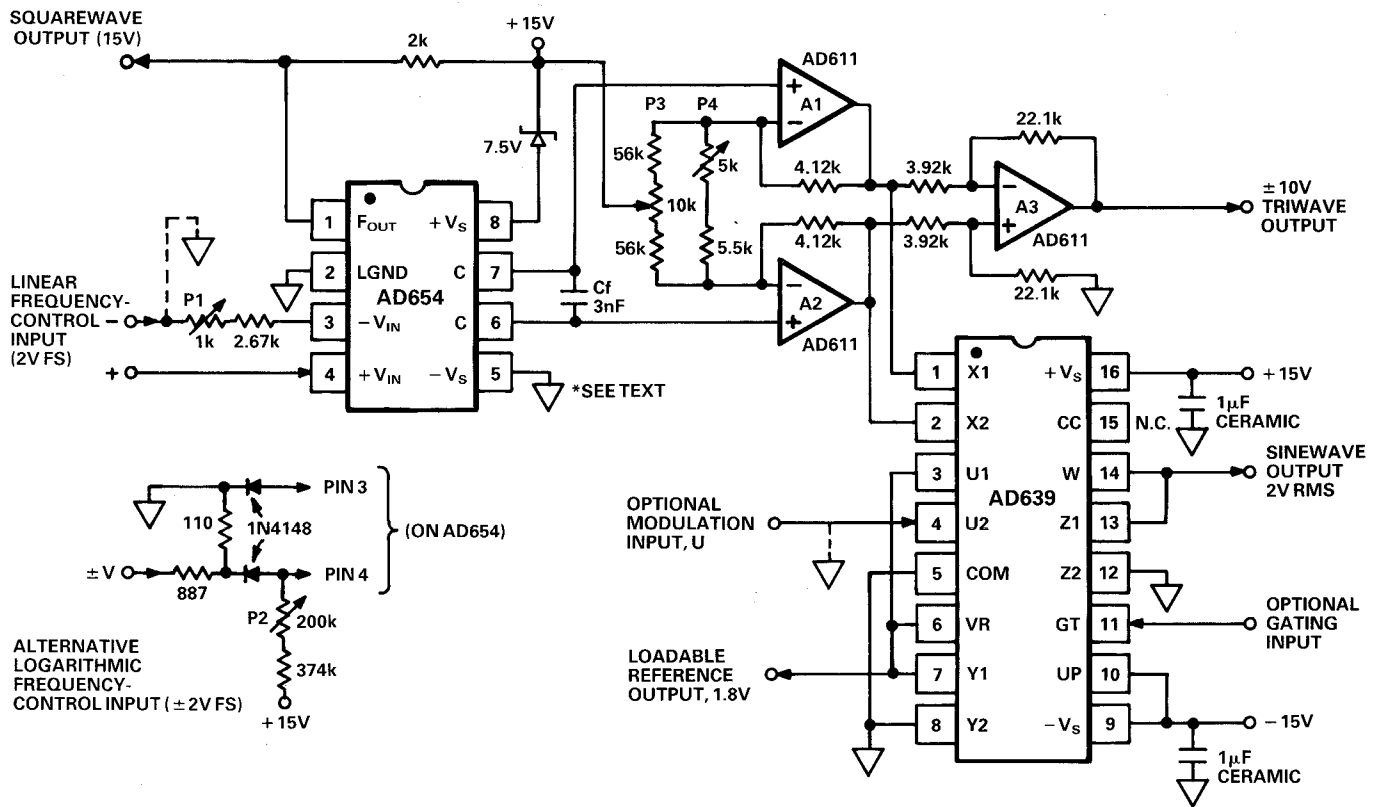
Figure 12. General-Purpose Function Generator

Applications

WIDE-RANGE WAVEFORM GENERATOR

Figure 12 shows an inexpensive signal generator, providing voltage control of frequency from 20Hz to 20kHz and a pre-set sine amplitude of 2.8V (within 0.1dB of 2V rms). This output may be further modulated by an input of up to ±2.8V to U2, or gated off by an input of +1.5V or more to GT; Figure 13 shows the gated response. If required, a further input can be summed into Z2. The sine output can be set to 10V amplitude by connecting UP to VR and grounding U1.

An AD654 is used to generate the triwave which appears across the timing capacitor C_f, and is buffered, amplified and level-shifted by A1 and A2. Using a spectrum analyzer, P3 and P4 are adjusted to minimize even- and odd-harmonic distortion, respectively. The triwave linearity is not good enough to realize the inherent capabilities of the AD639, but total harmonic distortion is in the −50dB to −60dB range. A3 provides further gain for a ±10V triwave output. The square-wave output is taken directly from the AD654 and is unbuffered. It swings between ground and +15V; if pins 2 and 5 of the AD654 are connected to −15V, this output is 30V pk-pk.

The frequency scaling with the linear input (shown) is 10kHz/V, calibrated using P1. The frequency can be controlled manually, using a potentiometer and the V_r output of the AD639. P1 has sufficient trim range to provide a full-scale frequency of 20kHz with the 1.8V peak input. The alternative input scheme provides a "log-sweep" response with an approximate scaling of 10^V kHZ (when V is in volts). The range is now from about 10Hz to 100kHz; the frequency should be set to 1kHz with V = 0, using P2. The frequency is now sensitive to variations in
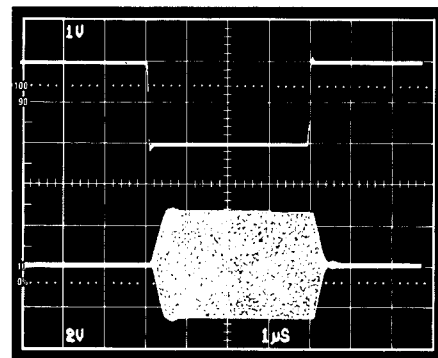

Figure 13. Gated Output. Top Trace: 0 to +2V Gate Input. Bottom Trace: 2V rms Gated Sine Output

both temperature and the +15V supply, but stability will be adequate for many applications.

Frequency Multiplication

Because of the exceptionally wide angular range of the numerator function of the AD639, it is possible to generate sinewave outputs with 2, 3, 4 or 5 times the triwave frequency using the cosine mode for even multiples or the sine mode for odd multiples.[2] For example, to multiply the output frequency by 3, use the sine function with the X-input driven to ±5.4V (±270°). Distortion remains low; all harmonics are typically under −50dBs, even for the frequency-quintupling mode.

[2]For full details see "A Remarkable Monolithic Microsystem Generates Trigonometric Functions," Barrie Gilbert, *Industrial Electronics Equipment Design*, September 1984, pp. 19-24. Reprints available.
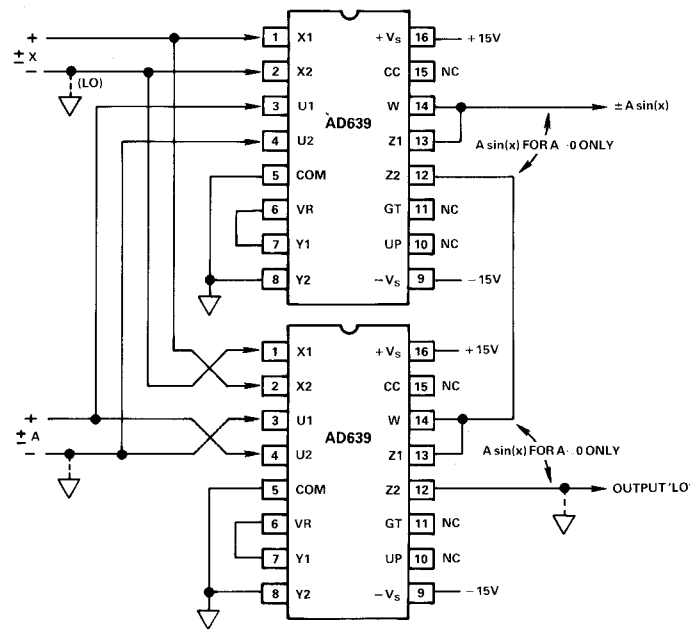
Figure 14. Four-Quadrant Sine Multiplier; for Cosine, Interchange X1 and X2 and Connect Angle 'Lo' Input to VR

FOUR-QUADRANT SINE/COSINE MULTIPLICATION

In synchro applications it is often necessary to multiply an AC sinusoidal 'carrier' by a further sine 'modulation' function. This can be achieved in two ways; the first is suitable only when there is a large ratio between the carrier frequency and the modulation frequency. Using a single AD639, the carrier input $A\sin(\omega t)$ is applied to U2, and a DC bias voltage established on U1 (which can be provided by a series resistor connected to $+V_s$). The modulation input, x, is applied to angle inputs connected for $-\sin(x)$. The output is then $W = -\sin(x)$ $(U_1 - A\sin(\omega t))$. Using AC-coupling to the load, the voltage $A\sin(x)\sin(\omega t)$ results. Since the peak value of W is $(U_1 + A)$, a maximum of about 6V amplitude can be achieved before output saturation. A further limitation of this approach is that the AC-coupling may allow excessive transmission of the sine modulation function. However, with typical values of 400Hz for the carrier and 10Hz for the upper modulation frequency, this simple approach is practical. Cosine modulation is similarly achieved.

An alternative method is DC-coupled and thus imposes no frequency-ratio limitations; it also allows an input/output amplitude of up to 12V. Two AD639s are used (Figure 14), the second having both the X- and U-interfaces phase-inverted relative to the first, and the two outputs are summed. The figure shows a general bipolar input, A, applied to the U-inputs. The first device generates $A\sin(x)$ when A is positive and zero when A is negative. The second device generates $-A\sin(x)$ (actually $A\sin(-x)$) when A is negative and zero when A is positive. The instantaneous sum of the two half-sines is $A\sin(x)$. The switching speed of the U-interface is adequate to handle a sinusoidal input $A = (10V)\sin(\omega t)$ at frequencies up to at least 1kHz, without significant crossover distortion. In synchro applications errors as small as 5 arc-minutes can be achieved.

Polar-to-Cartesian Conversion
Using a pair of AD639s connected as shown in Figure 14, and a second pair connected similarly for the cosine function, a vector of magnitude A and angle x can be resolved into its orthogonal components $A\sin(x)$ and $A\cos(x)$, with unrestricted operation in all quadrants and very high accuracy.

Cartesian-to-Polar Conversion
A point Z,U in a plane can be converted to a magnitude component, A, and an angle component, θ. A suitable vector summation circuit can be found in the AD637 data sheet. The AD639 in the arctangent mode can provide the angle output $\theta = \tan^{-1}(Z/U)$. If U is bipolar, an absolute-value circuit using an AD630 should be added.

Sine/Cosine (Quadrature) Oscillators

Quadrature oscillators generate a pair of sinusoidal outputs displaced by 90°, and invariably are based on a "state-variable" loop consisting of two integrators and a sign-inverter. Practical difficulties in this approach are (1) considerable additional circuitry is required to control the amplitude of the oscillation; (2) a trade-off arises between the settling-time of this control circuitry and the distortion level, particularly troublesome at low frequencies; (3) the amplitude balance of the two outputs is dependent on the matching of two time-constants; (4) two tracking analog multipliers or multiplying DACs are needed if the frequency is to be programmable.

These problems are avoided using a function-shaping technique based on a triwave oscillator, which requires only one time-constant, and whose frequency can thus be more easily controlled. The need for an amplitude control system is eliminated using the scheme shown in Figure 15. The two outputs have accurate amplitudes of 10V (without the need for an external reference source) or can be individually controlled by external voltages, without any effect on frequency. Variable-amplitude sine and cosine outputs can be added (using the Z-input discussed earlier) to provide continuously-variable phase-control of the output.
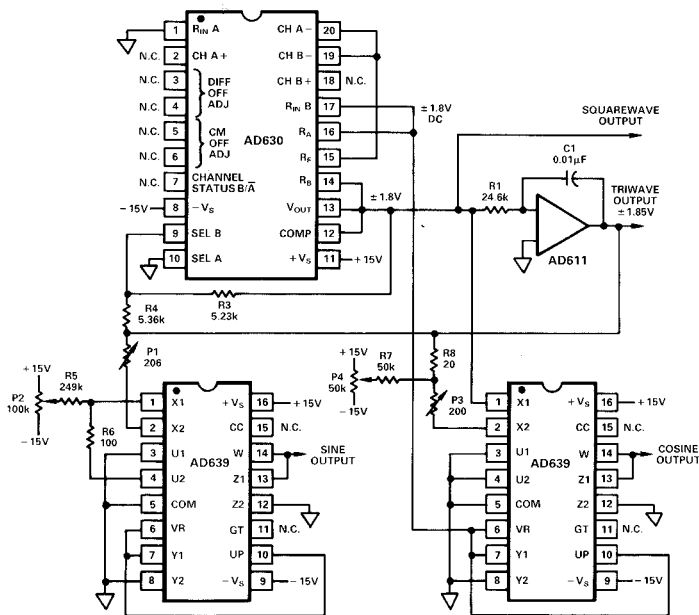
Figure 15. Quadrature Oscillator

The triwave oscillator comprises an AD630, which alternates the sign of the 1.8V reference from one of the AD639s to generate a square-wave output of ±1.8V amplitude, and an integrator formed by R1, C1 and the op-amp, which generates the triwave. The amplitude of the triwave is determined by the ratio of R3 and R4, and is nominally ±1.845V, 2.5% higher than needed at the inputs of the AD639s, providing the adjustment range needed to minimize distortion. In many applications, all adjustments can be eliminated; to do this, make $R3 = R4 = 5k\Omega$, omit P2, P4, R5 and R7 and replace P1, P3, R6, and R8 with short circuits. The frequency is nominally $1/4C_1R_1$, and is 1kHz with the component values shown. A variety of methods may be used to provide external control of frequency, including the use of another AD630 in series with R1, or the use of a multiplying DAC.
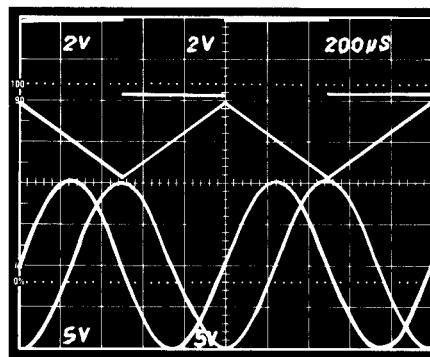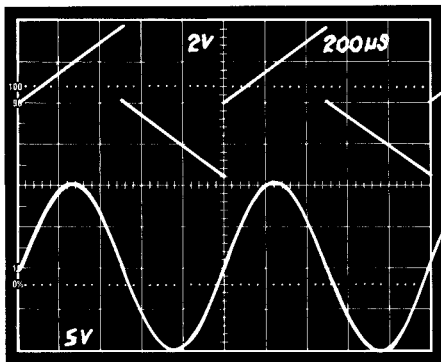

Figure 16. Top Waveform: Difference Voltage Between Triwave and Squarewave. Bottom Waveform: Resulting Output

The sine output is generated using the triwave directly. P1 and P2 should be adjusted using a spectrum analyzer for minimum odd-order and even-order harmonics, respectively. The cosine output is generated by using the *difference* between the triwave and the square-wave, shown in the upper waveform in Figure 16. This composite voltage first generates a sine-function over the range 0 to +180°, then over the range 0 to −180°, to produce the function shown in the lower waveform, which can be seen to be 90° out of phase with the triwave. The complete set of waveforms available from this generator are shown in Figure 17.

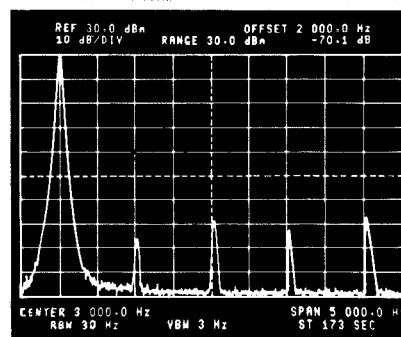Figure 17. Timing Relationships Between All Outputs of the Quadrature Oscillator

Figure 18. Spectrum of Cosine Output at 1kHz

P3 and P4 are adjusted for minimum odd-order and even-order cosine harmonics, respectively; Figure 18 shows the cosine spectrum for a well-adjusted circuit.

Due to the finite transition time back to the baseline in the drive voltage to the cosine generator, a brief spike occurs at the zero-crossing of this output. The frequency components will be beyond the bandwidth of the output amplifier in the AD639, and the energy contained in these spikes will not generally be troublesome. They may be further reduced, if necessary, by adding a capacitor between pins 14 and 15, to roll off the AD639 output response.